Subject Code:24BM11RC01 **R-24** **Reg No:** | | | | | | | | | | | | |

GAYATRI VIDYA PARISHAD COLLEGE OF ENGINEERING FOR WOMEN
(AUTONOMOUS)
(Affiliated to Andhra University, Visakhapatnam)

B.Tech. - I Semester Regular Examinations, December / January – 2025

CALCULUS AND DIFFERENTIAL EQUATIONS

(Common to All branches)

1. All questions carry equal marks
2. Must answer all parts of the question at one place

Time: 3Hrs. **Max Marks: 70**

UNIT-I

1. a. If u = log($x^3 + y^3 + z^3$ -3xyz) then show that $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$. [7M]
   b. If u = $x^2 + y^2 + z^2$, v = xy + yz + zx, w = x + y + z, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$. Are u,v,w functionally related? If so find the relation between them. [7M]

OR

2. a. If u = $x^2 + y^2 + z^2$ and x = e^{2t} , y = e^{2t}cos3t, z = e^{2t}sin3t, find $\frac{du}{dt}$. [7M]
   b. If f(x,y) = $\tan^{-1}$(xy), compute f(0.9,-1.2) approximately using Taylor's series. [7M]

UNIT-II

3. a. Examine the function f(x,y)= $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ for extreme values. [7M]
   b. Divide 24 into three parts such that the continued product of the first, square of the second and the cube of the third may be maximum. [7M]

OR

4. a. Examine the function f(x,y)= $x^3 + y^3$ -3axy for maximum and minimum values. [7M]
   b. A unit sphere $x^2 + y^2 + z^2$ =1 is placed in a room where the temperature T at any point (x,y,z) is T = 400xyz^2. Determine the highest temperature on the surface of the unit sphere. [7M]

UNIT-III

5. a. Evaluate the integral by changing the order ofintegration $\int_0^1 \int_x^{\sqrt{x}} xy\,dy\,dx$ [7M]
   b. Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\,dz\,dx\,dy$ [7M]

OR

6. a.Determine the area lying inside the circle r = asinθ and outside the cardioidr = a(1-cosθ) using double integral. [7M]
   b. Find the volume cut off from the cylinder $x^2 + y^2$ = ax by the planes z = 0 and z = x. [7M]

UNIT-IV

7. a. Solve (y^4 + 2y)dx + ($xy^3 + 2y^4$ -4x) dy = 0. [7M]
   b. The normal temperature of the air is considered to be30°C in a room. Ifa body when placed in the room cools from 80° C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. [7M]

OR

8. a. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 3y = e^x \cos x + x$ [7M]
   b. Using the method of variation of parameters solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ [7M]

UNIT-V

9. a. Find the Laplace transform of the function f(t) = $e^{-t}\int_0^t \frac{sint}{t}dt$ [7M]

b. Solve the differential equation (D^3 – $3D^2$ +3D -1)y = $t^2e^t$where D = $\frac{d}{dt}$using Laplace transform if the initial conditions are $y\,(0) \;= 1, y'\,(0) \;=\; 0,\; y''(0) \;=\; -2.$ [7M]

OR

10. a. Evaluate $\int_0^\infty te^{-2t}sin3tdt$ using Laplace transform. [7M]

b. Applying convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$. [7M]